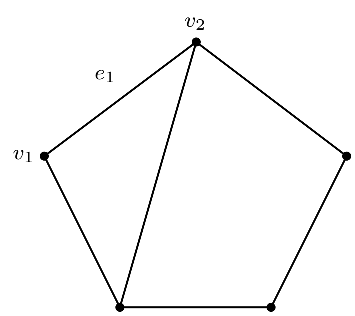
Graph Theory Definitions

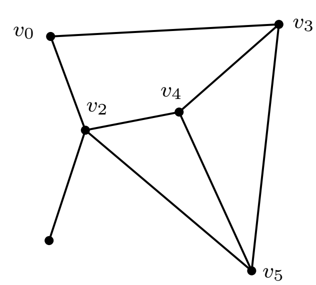
WDRP - Polyhedra

A **simple graph**, G, consists of a collection of points $V(G)$ called **vertices** (or **nodes**) and a collection of segments, $E(G)$, connecting vertices called the **edges** of G. In particular the edges are thought of as unordered pairs of vertices. For example, in the graph below, we write the edge $e_1 = (v_1, v_2) = (v_2, v_1)$. An edge between two vertices v, w is said to be **incident** to v and w. In the example below e_1 is incident to v_1 and v_2. The **degree** of a vertex is the number of edges incident to it. In the example below the degree of v_2 is 3 while the degree of v_1 is 2.



Remark. Consider the graph above. It's possible to think of the vertices as cities, and the edges between them direct highways connecting them. Similarly, the graph could also represent the connections between homes by telephone wires. Or even a group of students who have shared phone numbers. All this to say, similar to how numbers isolate the property of "amounts" of things, graphs do something similar when it comes to isolating the relational properties of a (complex) systems of objects.

Keeping with the idea of "depicting relationships between objects" the next definitions are natural sequels. A sequence of distinct edges where the vertices are not repeated (except the first and last) is called a **path**. It's easiest to just think of a path as walking around the graph from one vertex to another. If a path has at least one edge and the first and last vertices are the same then it is called a **cycle**. Again, think of this as doing a loop on the graph without walking backwards. In the graph below we have a cycle $v_0 \to v_3 \to v_4 \to v_2$, while $v_3 \to v_4 \to v_5$ is just a path.



Some final definitions. A graph which has no cycles is called a **tree**. Equivalently, a graph is a tree if there is only one unique path connecting two vertices. Why are these equivalent? A graph is **connected** if for any two vertices, there exists a path between them. Otherwise it is called **disconnected**. The **components** of a graph are the vertices which have paths connecting them. For example, the graph below has 3 components, each of which is a tree. A disconnected graph where each component is a tree is called a **forest**. The vertices of degree 1 in a tree are known as **leaves**.